Charge and electric fields

Electric charge and coulomb's law

There is a property of nature that we call "charge." We can create objects that are charged by rubbing them with other objects. When we do this over and over again with many types of objects, we discover that there are only two types of charges. We conclude that there are only two types of charges by the following logic:

- First, we find that like charges repel on another.
- Second we find that some charged objects attract each other
- Finally, we find that if object A attracts both object B and object C, which means that B and C have a charge that is different than A, then B and C repel, and therefore must be like charges
- Therefore, charges are either like A or like B and C.

Charge is a fundamental property of the basic particles in atom, namely electrons, protons, and neutrons. The neutron has no charge, while the electron and proton have opposite but equal charge. In our units, the charge on the electron is -1.6x10^{-19}C and the charge on the proton is +1.90x10^{-19}C. Macroscopic objects become charged when they gain or loose electrons so that the positive charge of all the protons does not exactly balance the negative charge of all the electrons.

We can determine the form of the form between two charged objects by doing careful experiments in which we vary the distance between the objects and vary their charges. We find Coulomb's law, which describes the force between two point charges:

$$F = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}$$

where r is the separation of the point charges, and q_1 and q_2 are the amount of charge on the charges. The force is attractive if the charges are opposite (q_1q_2<0) and repulsive if the charges are like (q_1q_2.>0). If the charges are not point charges but are spherically symmetric, then r is the center-to-center spacing of the charges. The constant ε, which is called the permittivity of free space is equal to:

$$\varepsilon_o = 8.85\times10^{-12}\ \mathrm{C^2/Nm^2}\ .$$

We often define a constant *k* to describe the coulomb force:

$$k = \frac{1}{4\pi\varepsilon_o} = 8.99\times10^{8}\ \mathrm{Nm^2/C^2}$$

Let's do some examples

1. Compare the electric force between the electron and proton in the ground state of a hydrogen atom. Assume their separation is the Bohr radius = 0.59 x 10^{-10} m.

2. Consider two pith mass, each with a mass of 0.0001g that are hanging from very light threads that are attached to the ceiling 2.5 cm apart. Each thread is 6.0 cm, and they make an angle of 10° with the vertical. Calculate the charge on each pith ball, assuming the charges are equal in magnitude.

Electric fields

How is the force between two charged particles communicated? In Newton's time there was the idea of action at a distance. However, in the 19th century, Michael Faraday came up with the idea of the field. The present of charged objects changed the nature of the space around the charges. These changes to the space propagate out from the charges and then other charged objects that are nearby feel the changes the first charges induces in the space. This change in the space around charged particles is called an electric field, and we say that charged particles produce and electric field surrounding them, which other charged objects interact with. We define the electric field at any point in space in terms of the force that a "test" charge would experience if it were place at that point (and assuming the test charge doesn't cause any of the charges that created the electric field to move).

$$\vec{E} = \frac{\vec{F}}{q'}$$

where q' is the charge on the test charge. This is a definition. The field is the force per unit of charge. Note that the field is a vector field because the force is a vector, and the direction of the field is either the same or opposite to the direction of the force, depending on whether you used a positive or negative test charge. The units of electric field is N/C.

If I place an object with a charge q at a point in space where there is a field ***E***, then the force on that object (assuming it doesn't cause the charges that created the field to move) will be:

$$\vec{F} = q\vec{E}\,.$$

Example:
What is the acceleration of an electron that is placed in an electric field of 15.5 N/C?

We have actually been using the idea of a field in our work in mechanics without realizing it. When we right the force of gravity as:

$$F = mg\,.$$

This just says that the force of gravity is equal to it mass times the force per unit mass. The gravitational field strength on Earth is about 10 N/kg, which we note has the same units as m/s^2.

Example:
An electic field projectile. An oil drop with a charge of +1e and mass of $1.0\text{x}10^{-15}$ is traveling at a velocity of 100 m/s in the +x direction in a region where there is an electric field of 500 N/C in the +y direction. How far in the x direction has the oil drop traveled after its y position has changed by 0.05m?

Electric field of a point charge

Using the definition of electric field, we can now calculate the electric field in the vicinity of an isolated point charge of q. Let's place a point charge q at the origin. Now, we determine the electric field by bringing in a test charge and measuring the force on that test charge. If we bring the test charge q' and place it at position ***r***, then the force acting on the test charge will be:

$$\vec{F} = \frac{kqq'}{r^2}\hat{r}\,,$$

so using the definition of electric field we find that the electric field around a point charge is:

$$\vec{E} = \frac{kq}{r^2}\hat{r}\,.$$

Note that if the point charge is positive, the electric field is directed radially outward, and if q is negative, the field is directed radially inward.

Superposition and the field of a dipole

What happens to the electric field in the vicinity of multiple point charges. The answer is that we just have to add up the electric field due to each point charge and that is the total electric field. Don't forget, however, that we must add the vectors. To see this is the case, consider a test charge q' in the vicinity of two point charges q_1 and q_2.

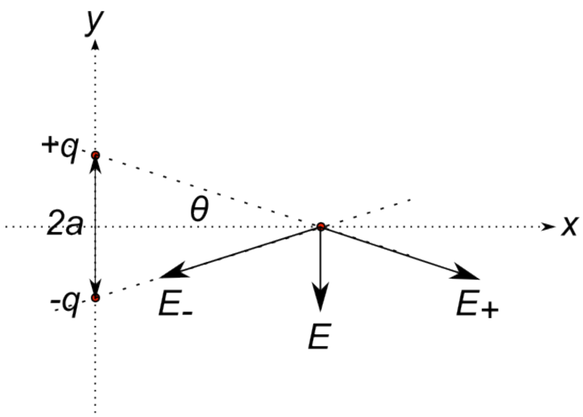
The resultant force on the test charge is:

$$\vec{F} = \vec{F}_1 + \vec{F}_2 = \frac{kq_1q_0}{r_{10}^2}\frac{\vec{r}_0 - \vec{r}_1}{r_{10}} + \frac{kq_2q_0}{r_{20}^2}\frac{\vec{r}_0 - \vec{r}_2}{r_{20}}$$

$$\vec{F} = q_0\left[\frac{kq_1}{r_{10}^2}\frac{\vec{r}_0 - \vec{r}_1}{r_{10}} + \frac{kq_2}{r_{20}^2}\frac{\vec{r}_0 - \vec{r}_2}{r_{20}}\right] = q_0\left[\vec{E}_1 + \vec{E}_2\right]$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

Example: A dipole is an arrangement of two equal and opposite charges $+q$ and $-q$, that are separated by a distance 2a. Assume the dipole is oriented along the y axis; determine the electric field at all points along the x axis.



From the figure, we see that the x-components of the fields from the two charges will cancel, and the y-components of the fields from the two charges are equal in magnitude. The electric field is therefore given by:

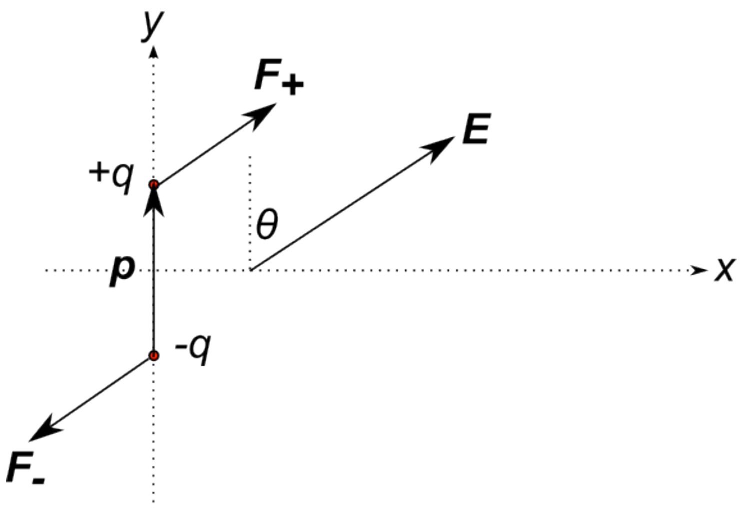
$$\vec{E} = \frac{-2kq\sin\theta}{\left(a^2 + x^2\right)}\hat{y} = \frac{-2kq}{\left(a^2 + x^2\right)}\frac{a}{\left(a^2 + x^2\right)^{1/2}}\hat{y} = \frac{-k(2aq)}{\left(a^2 + x^2\right)^{3/2}}\hat{y}$$

In the limit that we are far from the dipole, where x>>a, this result is approximately:

$$\vec{E} = \frac{-k2aq}{x^3}\hat{y}$$

The quantity $2aq$ is referred to as the dipole moment p. Actually, $\boldsymbol{p}$ is a vector, and the direction of the vector is from the positive charge to the negative charge.

If we place the dipole in a uniform electric field, the net force on the dipole will be zero, because the force on $+q$ is equal and opposite to the force on $-q$. However, the field can exert a torque on the dipole. Consider the following diagram. The torque on the dipole about the point at the center of the dipole is equal to:



$\tau = 2aqE\sin\theta$

where θ is the angle between the dipole vector and electric field vector. We see, therefore, that:

$\vec{\tau} = \vec{p} \times \vec{E}$

Field of continuous charge distributions

If we have a continuous charge distribution, then we must use integration to calculate the electric field

A ring of charge

A ring of charge +Q and radius a. The electric field along the axis of the ring (the x-axis) is:

$$\vec{E} = \frac{kqx}{\left(x^2 + a^2\right)^{3/2}}\hat{x}$$

Check the limits at x=0 and x>>a

A disk of charge

A disk of surface charge density σ and radius R. The electric field along the axis of the ring is:

$$\vec{E} = \frac{-\sigma}{2\varepsilon_o}\left[\frac{x}{\sqrt{x^2 + R^2}} - 1\right]$$

To do this, we need to evaluate the following integral:

$$\int \frac{rdr}{\left(x^2 + r^2\right)^{3/2}}$$