First, we will calculate the energy stored in a capacitor that has capacitance C and charge Q. The potential across the capacitor is

$$\Delta V = \frac{Q}{C}$$

The energy stored in the capacitor is equal to the work that is done by an outside force to assemble a charge $+Q$ on one plate and $-Q$ on the other. Consider the case where the capacitor has some charge q, and calculate how much work, dW, would be required to transfer an infinitesimal amount of charge dq from one plate to the other (thereby increasing the charge on the capacitor to $q+dq$).

Now integrate the quantity you found above from q=0 to q=Q to determine the total work to move charge Q from one plate to the other. Show that the total energy stored in the capacitor is equal to

$U = \frac{1}{2}\frac{Q^2}{C}$, which is also

$$\boxed{U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}Q\Delta V = \frac{1}{2}C\Delta V^2}$$

Next, consider the case of a parallel plate capacitor. Show that the energy stored is

$$U = \frac{1}{2}\varepsilon_0 AdE^2$$

where A is the area of the plates, d is their separation, and E is the magnitude of the electric field between the plates.

If we divide the energy stored in the capacitor by the volume between the plates (this is the volume occupied by the electric field), we find the energy density

$$\boxed{u_E = \frac{1}{2}\varepsilon_0 E^2}$$

We interpret this as the energy density in an electric field. It turns out that this is a general result.

We can understand the concept of energy density by thinking about the work that would be required to separate the plates. Picture the capacitor as starting with the plates, already charged, having zero separation. We could then calculate the energy stored in the capacitor by determining the work required to move one plate a distance d.

The force that we must apply to move one of the plates is equal to the force the other plate exerts on it, which is equal to the charge on the first plate times the electric field due to other plate. Show that this is equal to:

$$F = Q\frac{Q}{2A\varepsilon_0} = \frac{A\varepsilon_0}{2}E^2$$

So the work required to separate the plates is equal to:

$$W = \left(\frac{A\varepsilon_0}{2}E^2\right)d$$

This is the same result we got before. We can think of the energy stored in the electric field as building up as we pull the plates apart. We can also think of the energy density in the field as the electrostatic force per unit area, or pressure, exerted by the electric field.